# Grade 6 Math Circles
March 5 & 6 & 7, 2024
Math Paradoxes - Problem Set

1. (**Coin Rotation Paradox**) Suppose that there is a circular loop with a coin sitting inside it, as shown in the diagram. The inner radius of the loop is two times the radius of the coin. Let the coin roll without slipping once around the loop. How many turns does the coin make relative to an external stationary observer?
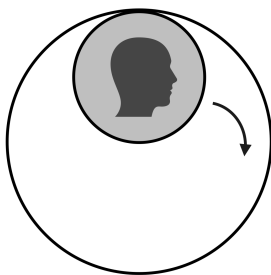
2. (**Coin Rotation Paradox**) *This problem is taken from 2015 AMC 10A Problem 14.* The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock at 12 o'clock. The disk has an arrow painted on it, initially point in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent (i.e., touching) when the arrow is next pointing in the upward vertical direction?
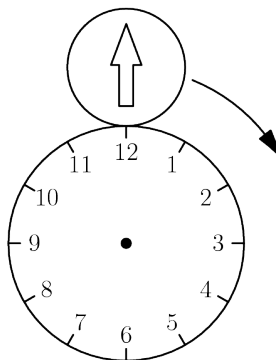
3. (**Will Rogers Phenomenon**) At a university, the numerical grades (out of 100) of two groups of students, R and S, are shown below.

$$R \text{ grades} = \{43, 56, 77, 78, 81, 88, 90, 95\}$$
$$S \text{ grades} = \{65, 70, 78, 82, 83, 85, 86, 88, 90, 94, 99, 100\}$$

(a) Calculate the mean of the grades of the students in R and the mean of the grades of the students in S. You may use a calculator, if you wish.

(b) Suppose that one student in group S is to move to group R. In the list above of S grades, circle all the grades that could be moved into R grades such that the mean of both lists increase.

4. (**Simpson's Paradox**) *This problem is adapted from an online blog post based on a news article*[1]. In a fictional country, all adults belong to one of the following groups based on their educational level:

- high school dropouts
- high school graduates with no post-secondary education
- people with some post-secondary education
- people with Bachelor's or higher degrees

Suppose that the median wage for the country has risen by 1%. However, the median wage within each group has all decreased.

(a) Explain why this is an example of Simpson's Paradox.

(b) The table below shows the percentage of the population belonging to each group before and after the change in median wage was measured. Based on this data, explain how the overall median wage of the population is seen to increase despite the drop in the median wage within all four groups.

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[1]**Source:** https://blog.revolutionanalytics.com/2013/07/a-great-example-of-simpsons-paradox.html

Educational Attainment	Proportion of Country Before	Proportion of Country After
high school dropouts	9%	9%
high school graduates with no post-secondary education	40%	40%
people with some post-secondary education	40%	20%
people with Bachelor's or higher degrees	12%	32%

5. (**Birthday Problem**) If there are $n = 3$ people in a room, how many pairs of people would you have to check to be sure that no one shares a birthday? How many pairs of people would you have to check if there are $n = 4$ people in the room? What about $n = 23$? Justify your answer.